Bursty activity in coupled networks

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INTRODUCTION

Our daily activity patterns are affecting the dynamics of social phenomena such as the spreading of epidemics, innovations and cooperation. Until recently, most epidemic models used Poisson processes to model human activity, where individual's activities are randomly distributed in time. But recent studies show that various human activity patterns, such as email and phone communications, web browsing and stock trading are characterized by bursts of rapidly occurring events separated by long periods of inactivity [1]. In other words, the time between events is heavy-tailed distributed and not exponentially distributed as in the case of a Poisson process. This deviation gives rise to significantly larger prevalence decay times than the ones predicted by the standard Poisson process based models [2, 3].

But what happens when a human is involved in more than one network, as is mostly the case in the real-world? For example, is a burst of activities in one social network synchronized with a burst of activities in another network? If a person is now answering all the emails received in the last hour, does it mean they are also going to post messages on Facebook at the same time? Or are different activities happening at separate bursts? And how does it affect the spreading dynamics in the networks? Intuitively one might imagine that the dynamics will be significantly changed by the nterleaving of non-Poisson processes.

Here we study the susceptible-infected (SI) model on a duplex consisting of two random networks, where similarly to [3], a susceptible node that becomes infected at time t, infects all its neighbors at time $t' > t$, where $t' - t$, the inter-event time, is drawn from a power law distribution with exponent 1 followed by an exponential cutoff

$$P(\tau) = \tau^{-1} \exp \frac{-\tau}{\tau_A} \tag{1}$$

We compare between the cases where a node infects all its neighbors from both networks after the same waiting time since becoming infected (*synchronized*), and where two different inter-event times are randomly chosen (*non-synchronized*).

RESULTS

Figure 1 shows the distribution of the time to full prevalence obtained for a duplex consisting of two Barabási-Albert (BA) networks of size $N = 10^5$ and average connectivity 2 (thus resulting in a duplex with average connectivity ≈ 4), when coupled randomly(1(a), 1(d)), positively (1(b), 1(e)) and negatively(1(c), 1(f)). Positive (negative) coupling means that nodes are sorted according to their degree and two nodes from each network are matched in order (in opposite order) of the degree rank. Inter-event times are drawn from either exponential or power-law distribution (as in Eq. (1) with average inter-event time equal to 58 ± 1 (resulting from a cutoff of $\tau_A = 300$). Different cutoffs (100, 500 and 800) as well as larger average connectivity (4) give qualitatively similar results (not shown here). Note that the average connectivities considered are quite small, as in datasets used to study bursty activity [2, 3].

In all 3 cases (random, positive and negative coupling), it is very clear that the synchronization has a much bigger effect in the case of power-law inter-event distribution. This makes sense since the exponentially distributed inter-event time is relatively homogenous, and therefore drawing the same waiting time or two different ones has a smaller effect, than in the case of heterogeneous inter-event times.

DISCUSSION

We have studied the effects of bursty activity patterns on spreading processes on coupled networks. Deviations from the standard Poisson model of inter-event times clearly and significantly affect decay time when an individual is part of more than network, with qualitatively similar results obtained for various types of couplings.

These results highlight once again the importance of considering the dependency and coupling between networks when studying network phenomena [4]. This applies both to the study of human processes such as disease spreading and social interactions, but also to traffic systems, communications systems, and the internet: the coupling of bursty activity changes the observed phenomena significantly, and must be accounted for clearly in formulating models.

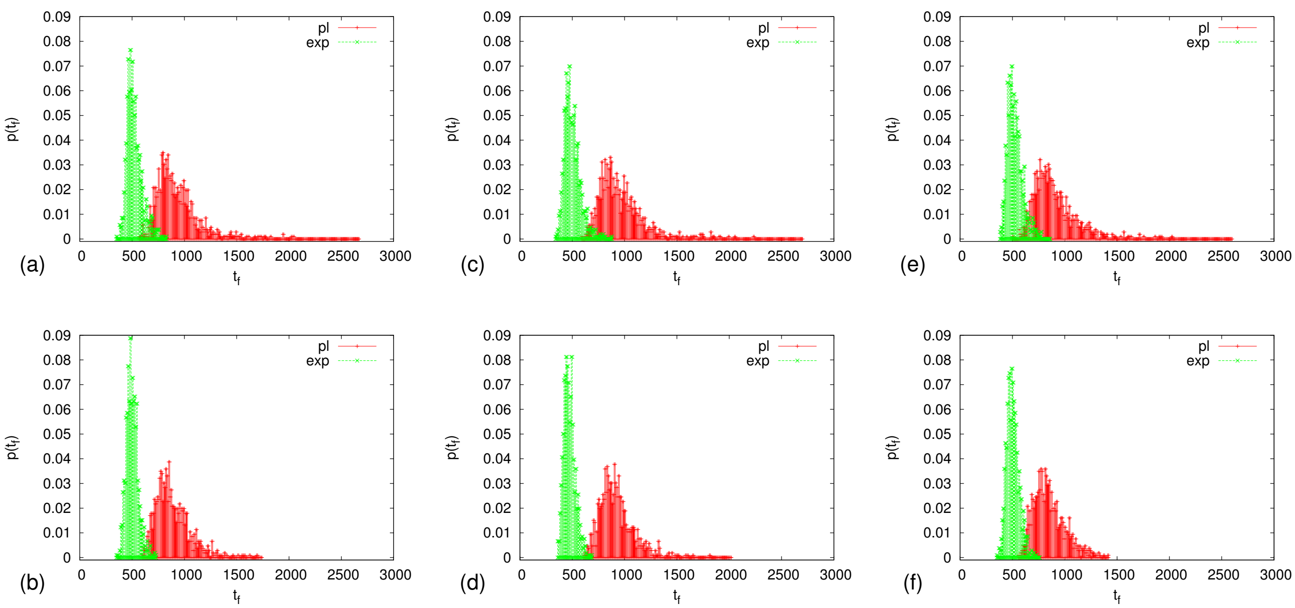

FIG. 1. Distribution of full prevalence times (t_f) where inter-event distribution is randomly drawn from an exponential distribution (green) or heavy-tailed distribution (red) with average equals to 58 ± 1. Different prevalence times obtained by 1000 computer simulations of a duplex consisting of two BA networks of size $N = 10^5$ and mean degree 2. (a) Synchronized infection activity on randomly coupled networks; (b) Non-synchronized infection activity on randomly coupled networks; (c) Synchronized infection activity on positively coupled networks; (d) Non-synchronized infection activity on positively coupled networks; (e) Synchronized infection activity on negatively coupled networks; (f) Non-synchronized infection activity on negatively coupled networks;

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