Modelling urban networks: some results and their limitations

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Introduction

What this talk is about

- How is human movement in an urban setting conditioned by the topology of the transportation networks?
- Are there any features common between large cities?
- How do quantitative results relate to the practice of urban planning?

My aim

- Present recent results on urban transportation, joint work with Saray Shai, Emanuele Strano, and Marc Barthélemy
- Real-world data driving a study using network science
- The limitations we hit when theory met practice
Complex networks
Networks and processes

Living between regularity and randomness \(^1\)

- Heterogeneous degree distribution, fragile notion of “neighbourhood”
- Evaluate processes at each node, affecting behaviour of neighbours, often with a stochastic component
- Canonical example is the SIR model of disease propagation

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A *multiplex* (or more correctly a *multilayer network*) is a collection of two (or more) networks:\(^2\)

- Nodes in the different networks are *coupled*
- Study properties of the individual networks or of the ensemble
- One network may be “less wide” than the other, and so offer “shortcuts” for processes

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Planarity limits permissible topologies

- Network embeds into Euclidean 2-space ($\mathbb{R}^2$)
- No crossings: all intersections form junctions
- (Doesn’t work precisely for all cities, e.g., Edinburgh, which have significant 3D structure)
- Limits the possibility for long-distance connections
- Typically quite modular, with highly-connected locales

Spatial multilayer networks

- Each layer is planar, but the multiplex typically isn’t

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Urban networks
Cities of different sizes and complexities
The problem: Urban transportation

Coupled transport networks

- Street and tube/subway form a multilayer network
- How does the addition of the tube affect travel times?
- How does this change as the tube speeds up?

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Framing the problem

Our study

- Simplify to treat as a purely topological problem
- Don’t model traffic congestion *per se*

Metrics

- Impact of tube speed on network usage, travel costs, and shortest paths
- Study the *betweenness centrality* of nodes as the relative speeds of the two networks changes
- How does *outreach* change?

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Methodology

Topological properties

- Compute metrics between all pairs \((i, j)\) of nodes in the street network \(V_s\)
- Compute ratio of metrics between travel using the streets only \textit{versus} using the whole multiplex

Geographical properties

- Network is spatial, so nodes have location in space, and a distance \(d(i, j)\) between pairs of street nodes
- Often scale distances according to network diameter, \(\frac{d(i,j)}{\sqrt{A}}\)
- Compare network metrics to geographical distances
Acquired street and tube data from Open Street Map

- Street network consisting of $v \in V_s$ nodes
- Tube network $V_t$
- Coupled at access points to form a multiplex

<table>
<thead>
<tr>
<th></th>
<th>$N_s$</th>
<th>$N_t$</th>
<th>Street diameter</th>
<th>Tube diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>London</td>
<td>325K</td>
<td>263</td>
<td>89km</td>
<td>60km</td>
</tr>
<tr>
<td>New York</td>
<td>68K</td>
<td>454</td>
<td>55km</td>
<td>57km</td>
</tr>
</tbody>
</table>
Data hygiene

Needed substantial manual cleaning
- Streets don’t meet, tubes don’t come up where they should, . . .

Choices to be made
- Tubes sometimes emerge mid-street, not at a junction
- Add a pseudo-junction for the tube to be coupled to
- Couple to junction at one end of the street or the other
- Do these choices make a difference?
Setting up the study – 2

Travel costs

- $\tau_s(i,j)$ the travel cost (in time units) between $i,j \in V_s$ using only street edges
- $\tau_m(i,j)$ the travel cost using the multiplex (street and tube)
- $0 \leq \beta \leq 1$ the ratio of speed between street and tube (tube is faster for smaller $\beta$)

Shortest paths

- $\sigma_{i,j}$ the number of shortest paths between $i,j \in V_s$ using only the street network
- Similarly define $\sigma_{i,j}^m$ the number of shortest paths using the multiplex
How much does the tube affect travel costs?

Metric

- Ratio of travel costs from a node \( i \in V_s \) to all other nodes using the multiplex vs using the streets only

\[
q_{ms}(i) = \frac{1}{N_s - 1} \sum_{j \in V_s} \frac{\tau_m(i, j)}{\tau_s(i, j)}
\]

Impact

- Halving \( \beta \) reduces \( \langle q_{ms} \rangle \) by about 20%
- Most journeys have a large street component that can’t be removed
Interdependence

Ratios of shortest paths

- **Inter-modal connectivity**
- \( \lambda(i, j) = \frac{\sigma_{i,j}^{m}}{\sigma_{i,j}} \)
- For \( \beta = 0.8 \), \( \langle \lambda \rangle = 0.7 \): 70% of journeys use the tube

Compare to scaled distances

- Scale based on \( \frac{d(i,j)}{\sqrt{A}} \)
- \( Q_\lambda(d) = \frac{1}{N(d)} \sum_{i,j \in V_s} \mathbb{1}_{d(i,j)=d} \lambda(i, j) \)
- Spatially short journeys benefit from hopping on the tube
Outreach

- **Spatial outreach** of a node $i$ is the average Euclidean distance to all nodes reachable with a travel cost $\tau$
- \[ L_\tau(i) = \frac{1}{N(\tau)} \sum_{j \in \{k | \tau_m(i, k) \leq \tau\}} d(i, j) \]
- How “commutable” is a city

![Graphs showing spatial outreach](images)
Betweenness centrality

Metric

- Compute $\sigma_{i,j}^m(v)$ the fraction of shortest paths that go through $v \in V_s$
- $bc_m(v) = \frac{1}{(N_s-1)(N_s-2)} \sum_{i,j \in V_s} \frac{\sigma_{i,j}^m(v)}{\sigma_{i,j}^m}$

Impact

- Shift congestion from roads to nodal points of tubes as $\beta$ decreases
- Tubes “decentralise” congestion to the ends of lines
- Betweenness centrality doesn’t move to the centre, as happens with mesh networks
Shifting spatial patterns

Examining the BC Gini as a function of $\beta$ and the interdependency $l$ in London (figure 7), we observe a non-trivial optimal value for $\beta$ for which flows are the most homogeneously distributed across street junctions. In New York (figure 7b), however, there seem to be room for small $\beta$ and small congestion and the absence of a non-trivial optimum for New York suggests (as discussed theoretically in [37]) that—surprisingly—it has a more marked monocentric aspect than London. In other words, the congestion in central places in New York is so large that introducing an efficient subway system is always better, even if it creates congestion at other points. Remarkably, these results on the BC and on the existence of an optimal point are thus in agreement with a recent theoretical model of coupled transportation networks, where—depending on the distribution of trip targets—one in which the optimal coupling is trivially the maximum, and another where a non-trivial optimal coupling exists [37].
Limitations
Topography only

No congestion “agents”

- Not modelling the traffic *per se*
- When we suck traffic into the tube, we assume that we *can*
- Either roads are “sufficiently big” or traffic “sufficiently light” – neither of which is actually the case

A more detailed model

- Make cost dependent on centrality?
- Limit capacity of edges?
Wrong metrics

Betweenness centrality is all-to-all

- Shortest paths between all pairs $i$ and $j$

A commuter model would capture which routes were more important

- Probabilistically weight the routes that people actually use
- Drive from real data, *i.e.*, TfL turnstile measurements
- Recent results show we can estimate route weights from census data on living and working population densities
Universality

Want to know that ideas work everywhere

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Getting more towards universality

- Can we synthesise cities with realistic (coupled) topologies?
- Generate plausible alternative topologies to explore
How do cities form and evolve?

- Villages coalesce over time, interconnections grow, . . .
- Certain topological structures seem to be very persistent over time
Faking it

How do cities form and evolve?

- Villages coalesce over time, interconnections grow, . . .
- Certain topological structures seem to be very persistent over time

Study historical events, for example the Black Death

- A combination of disease, healthcare, and social structure
- Is breaking one of these features sufficient to stop an epidemic?
- Do network features make some modern epidemics worse?
Conclusion

- Realistic to study urban-scale networks computationally using network science
- Data is publicly available, but needs care and cleaning
- Topology-driven analysis still shows useful results
- A commuter model would be useful, and seems to be possible from observed patterns (in London at least)
- Universal results are elusive and would require significant advances in synthetic urban network generation
Thank you

Topology! The stratosphere of human thought! In the twenty-fourth century it might possibly be of use to someone. . .

Alexander Solzhenitsyn, The First Circle
References


